MECHANICAL ENGINEERING PRINCIPLES

John Bird & Carl Ross

THIRD EDITION

Mechanical Engineering Principles

Third Edition

Why are competent engineers so vital?

Engineering is among the most important of all professions. It is the authors' opinions that engineers save more lives than medical doctors (physicians). For example, poor water, or the lack of it, is the second largest cause of human death in the world, and if engineers are given the 'tools', they can solve this problem. The largest cause of human death is caused by the malarial mosquito, and even death due to malaria can be decreased by engineers – by providing helicopters for spraying areas infected by the mosquito and making and designing medical syringes and pills to protect people against catching all sorts of diseases. Most medicines are produced by engineers! How does the engineer put 1 mg of 'medicine' precisely and individually into millions of pills, at an affordable price?

Moreover, one of the biggest contributions by humankind was the design of the agricultural tractor, which was designed and built by engineers to increase food production many-fold, for a human population which more-or-less quadruples every century! It is also interesting to note that the richest countries in the world are very heavily industrialized. Engineers create wealth! Most other professions don't!

Even in blue sky projects, engineers play a major role. For example, most rocket scientists are chartered engineers or their equivalents and Americans call their chartered engineers (and their equivalents), scientists. Astronomers are space scientists and not rocket scientists; they could not design a rocket to conquer outer space. Even modern theoretical physicists are mainly interested in astronomy and cosmology and also nuclear science. In general a theoretical physicist cannot, without special training, design a submarine structure to dive to the bottom of the Mariana Trench, which is 11.52 km or 7.16 miles deep, or design a very long bridge, a tall city skyscraper or a rocket to conquer outer space. It may be shown that the load on a submarine pressure hull of diameter 10 m and length 100 m is equivalent to carrying the total weight of about 7 million London double-decker buses!

This book presents a solid foundation for the reader in mechanical engineering principles, on which s/he can safely build tall buildings and long bridges that may last for a thousand years or more. It is the authors' experience that it is most unwise to attempt to build such structures on shaky foundations; they may come tumbling down – with disastrous consequences.

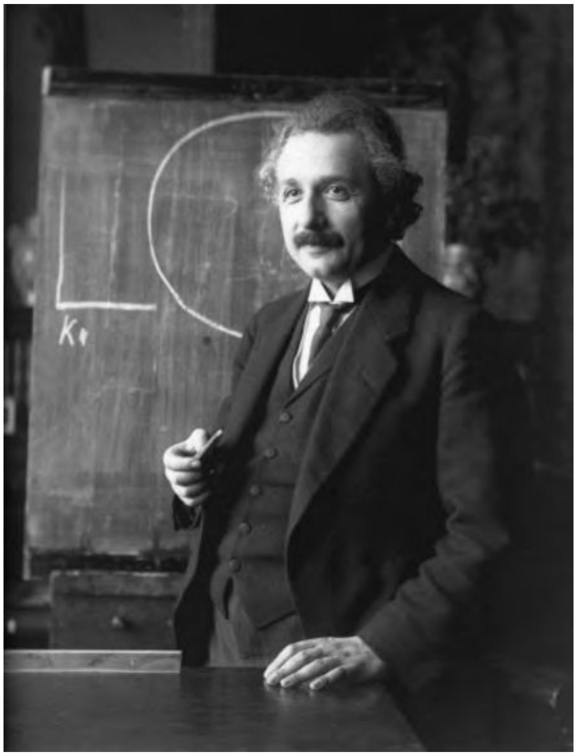
John Bird is the former Head of Applied Electronics in the Faculty of Technology at Highbury College, Portsmouth, U.K. More recently, he has combined freelance lecturing at the University of Portsmouth, with Examiner responsibilities for Advanced Mathematics with City and Guilds, and examining for the International Baccalaureate Organisation. He is the author of over 125 textbooks on engineering and mathematical subjects with worldwide sales of one million copies. He is currently a Senior Training Provider at the Defence School of Marine Engineering in the Defence College of Technical Training at H.M.S. Sultan, Gosport, Hampshire, U.K.

Carl Ross gained his first degree in Naval Architecture, from King's College, Durham University; his PhD in Structural Engineering from the Victoria University of Manchester; and was awarded his DSc in Ocean Engineering from the CNAA, London. His research in the field of engineering led to advances in the design of submarine pressure hulls. His publications and guest lectures to date exceed some 290 papers and books, etc., and he is Professor of Structural Dynamics at the University of Portsmouth, UK.

See Carl Ross's website below, which has an enormous content on science, technology and education.

http://tiny.cc/6kvqhx

Some quotes from ***Albert Einstein*** (14 March 1879–18 April 1955)



‘Scientists investigate that which already is; Engineers create that which has never been’

‘Imagination is more important than knowledge. For knowledge is limited to all we now know and understand, while imagination embraces the entire world, and all there ever will be to know and understand’

‘Everybody is a genius. But if you judge a fish by its ability to climb a tree, it will live its whole life believing that it is stupid’

‘To stimulate creativity, one must develop the childlike inclination for play’

Mechanical Engineering Principles

Third Edition

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Contents

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Preface

Mechanical Engineering Principles 3rd Edition aims to broaden the reader's knowledge of the basic principles that are fundamental to mechanical engineering design and the operation of mechanical systems.

Modern engineering systems and products still rely upon static and dynamic principles to make them work. Even systems that appear to be entirely electronic have a physical presence governed by the principles of statics.

In this third edition of *Mechanical Engineering Principles*, a further chapter has been added on revisionary mathematics; it is not possible to progress in engineering studies without a reasonable knowledge of mathematics, a fact that soon becomes obvious to both students and teachers alike. It is therefore hoped that this further chapter on mathematics revision will be helpful and make engineering studies more comprehensible. Minor modifications, some further worked problems, a glossary of terms and famous engineers' biographies have all been added to the text.

More has been added to the website for this new edition – such as full solutions being made available to both students and staff, and much more besides – see page x.

For clarity, the text is divided into **four sections**, these being:

Part 1 Revision of Mathematics
Part 2 Statics and Strength of Materials
Part 3 Dynamics
Part 4 Heat Transfer and Fluid Mechanics

***Mechanical Engineering Principles 3rd Edition* is suitable for the following:**

(i) National Certificate/Diploma courses in Mechanical Engineering
(ii) Undergraduate courses in Mechanical, Civil, Structural, Aeronautical & Marine Engineering, together with Naval Architecture
(iii) Any introductory/access/foundation course involving Mechanical Engineering Principles at University, and Colleges of Further and Higher education.

Although pre-requisites for the modules covered in this book include Foundation Certificate/diploma, or similar, in Mathematics and Science, **each topic considered in the text is presented in a way that assumes that the reader has little previous knowledge of that topic.**

Mechanical Engineering Principles 3rd Edition contains over **400 worked problems**, followed by over **700 further problems** (all **with answers**). The further problems are contained within some **150 Exercises**; each Exercise follows on directly from the relevant section of work, every few pages. In addition, the text contains **298 multiple-choice questions** (all **with answers**), and **260 short answer questions**, the answers for which can be determined from the preceding material in that particular chapter. Where at all possible, the problems mirror practical situations found in mechanical engineering. **387 line diagrams** enhance the understanding of the theory.

At regular intervals throughout the text are some **9 Revision Tests** to check understanding. For example, Revision Test 1 covers material contained in Chapter 1, Test 2 covers the material in Chapter 2, Test 3 covers the material in Chapters 3 to 6, and so on. No answers are given for the questions in the Revision Tests, but an **Instructor's guide** has been produced giving full solutions and suggested marking scheme. The guide is offered online free to lecturers/instructors – see below.

At the end of the text, a list of relevant **formulae** is included for easy reference, together with a **glossary of terms.**

'**Learning by Example**' is at the heart of *Mechanical Engineering Principles, 3rd Edition*.

JOHN BIRD
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HMS Sultan, formerly
University of Portsmouth and
Highbury College, Portsmouth
CARL ROSS Professor, University of Portsmouth

Free Web downloads
The following support material is available from *http://www.routledge.com/cw/bird*

For Students:

1. Full worked solutions to all 700 further questions contained in the 150 Practice Exercises
2. A list of Essential Formulae
3. A full glossary of terms
4. Multiple-choice questions
5. Information on 20 Famous Engineers mentioned in the text
6. Video links to practical demonstrations by Professor Carl Ross **http://tiny.cc/6kvqhx**

For Lecturers/Instructors:

1– 6. As per students 1–6 above.
7. Full solutions and marking scheme for each of the 9 Revision Tests; also, each test may be downloaded for distribution to students.
8. All 387 illustrations used in the text may be downloaded for use in PowerPoint presentations.

Part One

Revision of Mathematics

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Chapter 1

Revisionary mathematics

***Why it is important to understand:* Revisionary mathematics**

Mathematics is a vital tool for professional and chartered engineers. It is used in mechanical & manufacturing engineering, in electrical & electronic engineering, in civil & structural engineering, in naval architecture & marine engineering and in aeronautical & rocket engineering. In these various branches of engineering, it is very often much cheaper and safer to design your artefact with the aid of mathematics – rather than through guesswork. 'Guesswork' may be reasonably satisfactory if you are designing an artefact similar to one that has already proven satisfactory; however, the classification societies will usually require you to provide the calculations proving that the artefact is safe and sound. Moreover, these calculations may not be readily available to you and you may have to provide fresh calculations, to prove that your artefact is 'roadworthy'. For example, if you design a tall building or a long bridge by 'guesswork', and the building or bridge do not prove to be structurally reliable, it could cost you a fortune to rectify the deficiencies. This cost may dwarf the initial estimate you made to construct these artefacts, and cause you to go bankrupt. Thus, without mathematics, the prospective professional or chartered engineer is very severely handicapped.

At the end of this chapter you should be able to:

- convert radians to degrees
- convert degrees to radians
- calculate sine, cosine and tangent for large and small angles
- calculate the sides of a right-angled triangle
- use Pythagoras' theorem
- use the sine and cosine rules for acute-angled triangles
- expand equations containing brackets
- be familiar with summing vulgar fractions
- understand and perform calculations with percentages
- understand and use the laws of indices
- solve simple simultaneous equations

1.1 Introduction

As highlighted above, it is not possible to understand aspects of mechanical engineering without a good knowledge of mathematics. This chapter highlights some areas of mathematics which will make the understanding of the engineering in the following chapters a little easier.

Mechanical Engineering Principles, Bird and Ross, ISBN 9780415517850

Part One

1.2 Radians and degrees

There are 2π radians or 360° in a complete circle, thus:

$$\boldsymbol{\pi} \textbf{ radians} = \textbf{180°} \qquad \text{from which,}$$

$$\textbf{1 rad} = \frac{\textbf{180°}}{\boldsymbol{\pi}} \quad \text{or} \quad \textbf{1°} = \frac{\boldsymbol{\pi}}{\textbf{180}}\textbf{ rad}$$

where $\pi = 3.14159265358979323846$ .... to 20 decimal places!

Problem 1. Convert the following angles to degrees correct to 3 decimal places:
(a) 0.1 rad (b) 0.2 rad (c) 0.3 rad

(a) $0.1\text{ rad} = 0.1\text{ rad} \times \frac{180°}{\pi\text{ rad}} = \mathbf{5.730°}$

(b) $0.2\text{ rad} = 0.2\text{ rad} \times \frac{180°}{\pi\text{ rad}} = \mathbf{11.459°}$

(c) $0.3\text{ rad} = 0.3\text{ rad} \times \frac{180°}{\pi\text{ rad}} = \mathbf{17.189°}$

Problem 2. Convert the following angles to radians correct to 4 decimal places:
(a) 5° (b) 10° (c) 30°

(a) $5° = 5° \times \frac{\pi\text{ rad}}{180°} = \frac{\pi}{36}\text{ rad} = \mathbf{0.0873\ rad}$

(b) $10° = 10° \times \frac{\pi\text{ rad}}{180°} = \frac{\pi}{18}\text{ rad} = \mathbf{0.1745\ rad}$

(c) $30° = 30° \times \frac{\pi\text{ rad}}{180°} = \frac{\pi}{6}\text{ rad} = \mathbf{0.5236\ rad}$

Now try the following Practice Exercise

Practice Exercise 1 Radians and degrees

1. Convert the following angles to degrees correct to 3 decimal places (where necessary):
   (a) 0.6 rad (b) 0.8 rad
   (c) 2 rad (d) 3.14159 rad
   [(a) 34.377° (b) 45.837° (c) 114.592° (d) 180°]

2. Convert the following angles to radians correct to 4 decimal places:
   (a) 45° (b) 90°
   (c) 120° (d) 180°
   [(a) $\frac{\pi}{4}$ rad or 0.7854 rad
   (b) $\frac{\pi}{2}$ rad or 1.5708 rad
   (c) $\frac{2\pi}{3}$ rad or 2.0944 rad
   (d) π rad or 3.1416 rad]

1.3 Measurement of angles

Angles are measured starting from the horizontal 'x' axis, in an **anticlockwise direction**, as shown by θ_1 to θ_4 in Figure 1.1. An angle can also be measured in a **clockwise direction**, as shown by θ_5 in Figure 1.1, but in this case the angle has a negative sign before it. If, for example, $\theta_4 = 300°$ then $\theta_5 = -60°$.

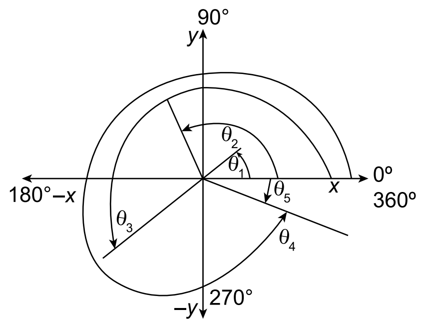


Figure 1.1

Problem 3. Use a calculator to determine the cosine, sine and tangent of the following angles, each measured anticlockwise from the horizontal 'x' axis, each correct to 4 decimal places:
(a) 30° (b) 120° (c) 250°
(d) 320° (e) 390° (f) 480°

(a) cos 30° = **0.8660** sin 30° = **0.5000**
tan 30° = **0.5774**

(b) cos 120° = **– 0.5000** sin 120° = **0.8660**
tan 120° = **– 1.7321**

(c) cos 250° = **– 0.3420** sin 250° = **– 0.9397**
tan 250° = **2.7475**

(d) cos 320° = **0.7660** sin 320° = **– 0.6428**
tan 320° = **– 0.8391**

(e) cos 390° = **0.8660** sin 390° = **0.5000**
tan 390° = **0.5774**

(f) cos 480° = **– 0.5000** sin 480° = **0.8660**
tan 480° = **– 1.7321**

These angles are now drawn in Figure 1.2. Note that cosine and sine always lie between –1 and +1 but that tangent can be >1 and <1

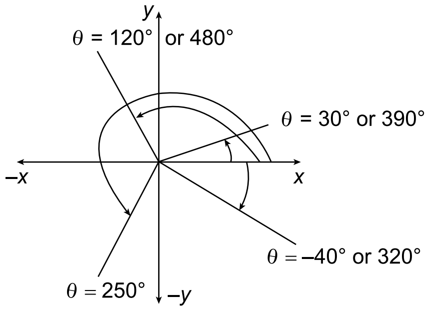


Figure 1.2

Note from Figure 1.2 that $\theta = 30°$ is the same as $\theta = 390°$ and so are their cosines, sines and tangents. Similarly, note that $\theta = 120°$ is the same as $\theta = 480°$ and so are their cosines, sines and tangents. Also, note that $\theta = -40°$ is the same as $\theta = +320°$ and so are their cosines, sines and tangents.
It is noted from above that

- in the **first quadrant**, i.e. where θ varies from 0° to 90°, all (A) values of cosine, sine and tangent are positive
- in the **second quadrant**, i.e. where θ varies from 90° to 180°, only values of sine (S) are positive
- in the **third quadrant**, i.e. where θ varies from 180° to 270°, only values of tangent (T) are positive
- in the **fourth quadrant**, i.e. where θ varies from 270° to 360°, only values of cosine (C) are positive

These positive signs, A, S, T and C are shown in Figure 1.3.

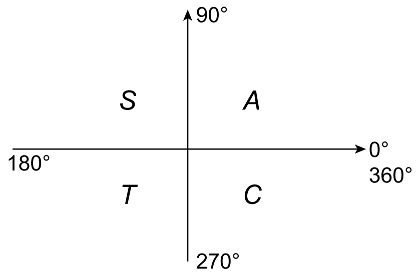


Figure 1.3

Now try the following Practice Exercise

Practice Exercise 2 Measurement of angles

1. Find the cosine, sine and tangent of the following angles, where appropriate each correct to 4 decimal places:
(a) 60° (b) 90° (c) 150°
(d) 180° (e) 210° (f) 270°
(g) 330° (h) – 30° (i) 420°
(j) 450° (k) 510°

[(a) 0.5, 0.8660, 1.7321
(b) 0, 1, ∞
(c) – 0.8660, 0.5, – 0.5774
(d) –1, 0, 0
(e) – 0.8660, – 0.5, 0.5774
(f) 0, –1, – ∞
(g) 0.8660, – 0.5000, – 0.5774
(h) 0.8660, – 0.5000, – 0.5774
(i) 0.5, 0.8660, 1.7321
(j) 0, 1, ∞
(k) – 0.8660, 0.5, – 0.5774]

1.4 Triangle calculations

(a) Sine, cosine and tangent

From Figure 1.4, $\sin\theta = \dfrac{bc}{ac}$ $\cos\theta = \dfrac{ab}{ac}$

$$\tan\theta = \frac{bc}{ab}$$

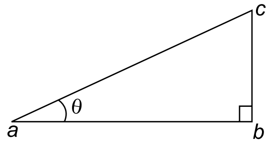


Figure 1.4

Problem 4. In Figure 1.4, if $ab = 2$ and $ac = 3$, determine the angle θ.

It is convenient to use the expression for cos θ, since 'ab' and 'ac' are given.

Hence, $\cos\theta = \frac{ab}{ac} = \frac{2}{3} = 0.66667$

from which, $\boldsymbol{\theta} = \cos^{-1}(0.66667) = \mathbf{48.19^\circ}$

Problem 5. In Figure 1.4, if $bc = 1.5$ and $ac = 2.2$, determine the angle θ.

It is convenient to use the expression for sin θ, since 'bc' and 'ac' are given.

Hence, $\sin\theta = \frac{bc}{ac} = \frac{1.5}{2.2} = 0.68182$

from which, $\boldsymbol{\theta} = \sin^{-1}(0.68182) = \mathbf{42.99^\circ}$

Problem 6. In Figure 1.4, if $bc = 8$ and $ab = 1.3$, determine the angle θ.

It is convenient to use the expression for tan θ, since 'bc' and 'ab' are given.

Hence, $\tan\theta = \frac{bc}{ab} = \frac{8}{1.3} = 6.1538$

from which, $\boldsymbol{\theta} = \tan^{-1}(6.1538) = \mathbf{80.77^\circ}$

(b) Pythagoras' theorem

Pythagoras' theorem* states that:
$(\text{hypotenuse})^2 = (\text{adjacent side})^2 + (\text{opposite side})^2$
i.e. in the triangle of Figure 1.5,

$$\mathbf{ac^2 = ab^2 + bc^2}$$

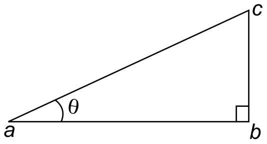


Figure 1.5

Problem 7. In Figure 1.5, if $ab = 5.1$ m and $bc = 6.7$ m, determine the length of the hypotenuse, ac.

From Pythagoras, $ac^2 = ab^2 + bc^2$

$= 5.1^2 + 6.7^2 = 26.01 + 44.89$

$= 70.90$

from which, $\boldsymbol{ac} = \sqrt{70.90} = \mathbf{8.42\ m}$

Now try the following Practice Exercise

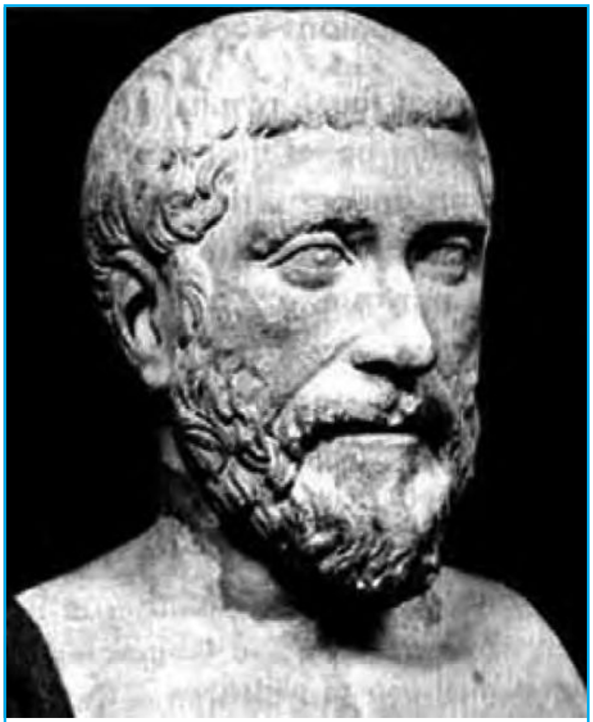
Practice Exercise 3 Sines, cosines and tangents and Pythagoras' theorem

In problems 1 to 5, refer to Figure 1.5.

1. If $ab = 2.1$ m and $bc = 1.5$ m, determine angle θ. [35.54°]
2. If $ab = 2.3$ m and $ac = 5.0$ m, determine angle θ. [62.61°]
3. If $bc = 3.1$ m and $ac = 6.4$ m, determine angle θ. [28.97°]
4. If $ab = 5.7$ cm and $bc = 4.2$ cm, determine the length ac [7.08 cm]
5. If $ab = 4.1$ m and $ac = 6.2$ m, determine length bc. [4.65 m]

(c) The sine and cosine rules

For the triangle ABC shown in Figure 1.6,



*Pythagoras of Samos (born approximately 570BC and died around 495BC) was an Ionian Greek philosopher and mathematician, best known for the Pythagorean Theorem. To find out more go to www.routledge.com/cw/bird

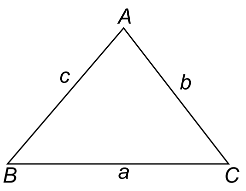


Figure 1.6

the sine rule states: $\frac{\boldsymbol{a}}{\sin \boldsymbol{A}} = \frac{\boldsymbol{b}}{\sin \boldsymbol{B}} = \frac{\boldsymbol{c}}{\sin \boldsymbol{C}}$

and the cosine rule states: $\boldsymbol{a^2 = b^2 + c^2 - 2bc\cos A}$

Problem 8. In Figure 1.6, if $a = 3$ m, $A = 20°$ and $B = 120°$, determine lengths b, c and angle C.

Using the sine rule, $\frac{a}{\sin A} = \frac{b}{\sin B}$

i.e. $\frac{3}{\sin 20°} = \frac{b}{\sin 120°}$

from which, $\boldsymbol{b} = \frac{3\sin 120°}{\sin 20°} = \frac{3 \times 0.8660}{0.3420}$

$= \mathbf{7.596\ m}$

Angle, $\boldsymbol{C} = 180° - 20° - 120° = \mathbf{40°}$

Using the sine rule again gives: $\frac{c}{\sin C} = \frac{a}{\sin A}$

i.e. $\boldsymbol{c} = \frac{a\sin C}{\sin A} = \frac{3 \times \sin 40°}{\sin 20°}$

$= \mathbf{5.638\ m}$

Problem 9. In Figure 1.6, if $b = 8.2$ cm. $c = 5.1$ cm and $A = 70°$, determine the length a and angles B and C.

From the cosine rule,

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 8.2^2 + 5.1^2 - 2 \times 8.2 \times 5.1 \times \cos 70°$$
$$= 67.24 + 26.01 - 2(8.2)(5.1)\cos 70°$$
$$= 64.643$$

Hence, **length,** $\boldsymbol{a} = \sqrt{64.643} = \mathbf{8.04\ cm}$

Using the sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B}$

i.e. $\frac{8.04}{\sin 70°} = \frac{8.2}{\sin B}$

from which, $8.04 \sin B = 8.2 \sin 70°$

and $\sin B = \frac{8.2\sin 70°}{8.04} = 0.95839$

and $\boldsymbol{B} = \sin^{-1}(0.95839) = \mathbf{73.41°}$

Since $A + B + C = 180°$, then

$\boldsymbol{C} = 180° - A - B = 180° - 70° - 73.41° = \mathbf{36.59°}$

Now try the following Practice Exercise

Practice Exercise 4 Sine and cosine rules

In problems 1 to 4, refer to Figure 1.6.

1. If $b = 6$ m, $c = 4$ m and $B = 100°$, determine angles A and C and length a.
   [$A = 38.96°$, $C = 41.04°$, $a = 3.83$ m]

2. If $a = 15$ m, $c = 23$ m and $B = 67°$, determine length b and angles A and C.
   [$b = 22.01$ m, $A = 38.86°$, $C = 74.14°$]

3. If $a = 4$ m, $b = 8$ m and $c = 6$ m, determine angle A. [28.96°]

4. If $a = 10.0$ cm, $b = 8.0$ cm and $c = 7.0$ cm, determine angles A, B and C.
   [$A = 83.33°$, $B = 52.62°$, $C = 44.05°$]

5. In Figure 1.7, PR represents the inclined jib of a crane and is 10.0 m long. PQ is 4.0 m long. Determine the inclination of the jib to the vertical (i.e. angle P) and the length of tie QR.

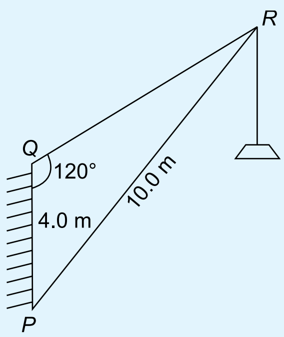


Figure 1.7

[$P = 39.73°$, $QR = 7.38$ m]

1.5 Brackets

The use of brackets, which are used in many engineering equations, is explained through the following worked problems.

Problem 10. Expand the bracket to determine A, given $A = a(b + c + d)$

Multiplying each term in the bracket by 'a' gives:

$$A = a(b + c + d) = \mathbf{ab + ac + ad}$$

Problem 11. Expand the brackets to determine A, given $A = a[b(c + d) - e(f - g)]$

When there is more than one set of brackets the innermost brackets are multiplied out first. Hence,

$$A = a[b(c + d) - e(f - g)] = a[bc + bd - ef + eg]$$

Note that $-e \times -g = +eg$

Now multiplying each term in the square brackets by 'a' gives:

$$\mathbf{A = abc + abd - aef + aeg}$$

Problem 12. Expand the brackets to determine A, given $A = a[b(c + d - e) - f(g - h\{j - k\})]$

The inner brackets are determined first, hence

$$A = a[b(c + d - e) - f(g - h\{j - k\})]$$
$$= a[b(c + d - e) - f(g - hj + hk)]$$
$$= a[bc + bd - be - fg + fhj - fhk]$$

i.e. $\mathbf{A = abc + abd - abe - afg + afhj - afhk}$

Problem 13. Evaluate A, given $A = 2[3(6 - 1) - 4(7\{2 + 5\} - 6)]$

$$A = 2[3(6 - 1) - 4(7\{2 + 5\} - 6)]$$
$$= 2[3(6 - 1) - 4(7 \times 7 - 6)]$$
$$= 2[3 \times 5 - 4 \times 43]$$
$$= 2[15 - 172] = 2[-157] = \mathbf{-314}$$

Now try the following Practice Exercise

Practice Exercise 5 Brackets

In problems 1 and 2, evaluate A

1. $A = 3(2 + 1 + 4)$ [21]
2. $A = 4[5(2 + 1) - 3(6 - 7)]$ [72]

Expand the brackets in problems 3 to 7.

3. $2(x - 2y + 3)$ $[2x - 4y + 6]$
4. $(3x - 4y) + 3(y - z) - (z - 4x)$ $[7x - y - 4z]$
5. $2x + [y - (2x + y)]$ [0]
6. $24a - [2\{3(5a - b) - 2(a + 2b)\} + 3b]$ $[11b - 2a]$
7. $ab[c + d - e(f - g + h\{i + j\})]$ $[abc + abd - abef + abeg - abehi - abehj]$

1.6 Fractions

An example of a fraction is $\frac{2}{3}$ where the top line, i.e. the 2, is referred to as the **numerator** and the bottom line, i.e. the 3, is referred to as the **denominator**.

A **proper fraction** is one where the numerator is smaller than the denominator, examples being $\frac{2}{3}, \frac{1}{2}, \frac{3}{8}, \frac{5}{16}$, and so on.

An **improper fraction** is one where the denominator is smaller than the numerator, examples being $\frac{3}{2}, \frac{2}{1}, \frac{8}{3}, \frac{16}{5}$, and so on.

Addition of fractions is demonstrated in the following worked problems.

Problem 14. Evaluate A, given $A = \frac{1}{2} + \frac{1}{3}$

The lowest common denominator of the two denominators 2 and 3 is 6, i.e. 6 is the lowest number that both 2 and 3 will divide into.

Then $\frac{1}{2} = \frac{3}{6}$ and $\frac{1}{3} = \frac{2}{6}$ i.e. both $\frac{1}{2}$ and $\frac{1}{3}$ have the common denominator, namely 6.

The two fractions can therefore be added as:

$$\mathbf{A} = \frac{\mathbf{1}}{\mathbf{2}} + \frac{\mathbf{1}}{\mathbf{3}} = \frac{3}{6} + \frac{2}{6} = \frac{3+2}{6} = \frac{\mathbf{5}}{\mathbf{6}}$$

Problem 15. Evaluate A, given $A = \frac{2}{3} + \frac{3}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is $3 \times 4 = 12$.

The two fractions can now be made equivalent, i.e. $\frac{2}{3} = \frac{8}{12}$ and $\frac{3}{4} = \frac{9}{12}$
so that they can be easily added together, as follows:

$$A = \frac{2}{3} + \frac{3}{4} = \frac{8}{12} + \frac{9}{12} = \frac{8+9}{12} = \frac{17}{12}$$

i.e. $\mathbf{A = \frac{2}{3} + \frac{3}{4} = 1\frac{5}{12}}$

Problem 16. Evaluate A, given $A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2}$

A suitable common denominator can be obtained by multiplying $6 \times 7 = 42$, because all three denominators divide exactly into 42.

Thus, $\frac{1}{6} = \frac{7}{42}$, $\frac{2}{7} = \frac{12}{42}$ and $\frac{3}{2} = \frac{63}{42}$

Hence, $A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2}$

$$= \frac{7}{42} + \frac{12}{42} + \frac{63}{42} = \frac{7+12+63}{42} = \frac{82}{42} = \frac{41}{21}$$

i.e. $\mathbf{A = \frac{1}{6} + \frac{2}{7} + \frac{3}{2} = 1\frac{20}{21}}$

Problem 17. Determine A as a single fraction, given $A = \frac{1}{x} + \frac{2}{y}$

A common denominator can be obtained by multiplying the two denominators together, i.e. xy

Thus, $\frac{1}{x} = \frac{y}{xy}$ and $\frac{2}{y} = \frac{2x}{xy}$

Hence, $A = \frac{1}{x} + \frac{2}{y} = \frac{y}{xy} + \frac{2x}{xy}$

i.e. $\mathbf{A = \frac{y+2x}{xy}}$

Note that addition, subtraction, multiplication and division of fractions may be determined using a **calculator** (for example, the CASIO fx-83ES or fx-991ES).

Locate the $\frac{\square}{\square}$ and $\square\frac{\square}{\square}$ functions on your calculator (the latter function is a shift function found above the $\frac{\square}{\square}$ function) and then check the following worked problems.

Problem 18. Evaluate $\frac{1}{4} + \frac{2}{3}$

(i) Press $\frac{\square}{\square}$ function
(ii) Type in 1
(iii) Press ↓ on the cursor key and type in 4
(iv) $\frac{1}{4}$ appears on the screen
(v) Press → on the cursor key and type in +
(vi) Press $\frac{\square}{\square}$ function
(vii) Type in 2
(viii) Press ↓ on the cursor key and type in 3
(ix) Press → on the cursor key
(x) Press = and the answer $\frac{11}{12}$ appears
(xi) Press $S \Leftrightarrow D$ function and the fraction changes to a decimal 0.9166666....

Thus, $\mathbf{\frac{1}{4} + \frac{2}{3} = \frac{11}{12}}$ $= \mathbf{0.9167}$ as a decimal, correct to 4 decimal places.

It is also possible to deal with **mixed numbers** on the calculator.

Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears

Problem 19. Evaluate $5\frac{1}{5} - 3\frac{3}{4}$

(i) Press Shift then the $\frac{\square}{\square}$ function and $\square\frac{\square}{\square}$ appears on the screen
(ii) Type in 5 then → on the cursor key
(iii) Type in 1 and ↓ on the cursor key
(iv) Type in 5 and $5\frac{1}{5}$ appears on the screen
(v) Press → on the cursor key
(vi) Type in - and then press Shift then the $\frac{\square}{\square}$ function and $5\frac{1}{5} - \square\frac{\square}{\square}$ appears on the screen
(vii) Type in 3 then → on the cursor key
(viii) Type in 3 and ↓ on the cursor key

(ix) Type in 4 and $5\frac{1}{5} - 3\frac{3}{4}$ appears on the screen

(x) Press = and the answer $\frac{29}{20}$ appears

(xi) Press $S \Leftrightarrow D$ function and the fraction changes to a decimal 1.45

Thus, $\mathbf{5\frac{1}{5} - 3\frac{3}{4} = \frac{29}{20} = 1\frac{9}{20}} = \mathbf{1.45}$ as a decimal.

Now try the following Practice Exercise

Practice Exercise 6 Fractions

In problems 1 to 3, evaluate the given fractions

1. $\frac{1}{3} + \frac{1}{4}$ $\left[\frac{7}{12}\right]$
2. $\frac{1}{5} + \frac{1}{4}$ $\left[\frac{9}{20}\right]$
3. $\frac{1}{6} + \frac{1}{2} - \frac{1}{5}$ $\left[\frac{7}{15}\right]$

In problems 4 and 5, use a calculator to evaluate the given expressions

4. $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21}$ $\left[\frac{1}{21}\right]$
5. $\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9}$ $\left[-\frac{9}{10}\right]$
6. Evaluate $\frac{3}{8} + \frac{5}{6} - \frac{1}{2}$ as a decimal, correct to 4 decimal places. $\left[\frac{17}{24} = 0.7083\right]$
7. Evaluate $8\frac{8}{9} \div 2\frac{2}{3}$ as a mixed number. $\left[3\frac{1}{3}\right]$
8. Evaluate $3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10}$ as a decimal, correct to 3 decimal places. [2.567]
9. Determine $\frac{2}{x} + \frac{3}{y}$ as a single fraction. $\left[\frac{3x + 2y}{xy}\right]$

1.7 Percentages

Percentages are used to give a common standard. The use of percentages is very common in many aspects of commercial life, as well as in engineering. Interest rates, sale reductions, pay rises, exams and VAT are all examples where percentages are used.

Percentages are fractions having 100 as their denominator.

For example, the fraction $\frac{40}{100}$ is written as 40% and is read as 'forty per cent'.

The easiest way to understand percentages is to go through some worked examples.

Problem 20. Express 0.275 as a percentage.

$$0.275 = 0.275 \times 100\% = \mathbf{27.5\%}$$

Problem 21. Express 17.5% as a decimal number.

$$17.5\% = \frac{17.5}{100} = \mathbf{0.175}$$

Problem 22. Express $\frac{5}{8}$ as a percentage.

$$\frac{5}{8} = \frac{5}{8} \times 100\% = \frac{500}{8}\% = \mathbf{62.5\%}$$

Problem 23. In two successive tests a student gains marks of 57/79 and 49/67. Is the second mark better or worse than the first?

$$57/79 = \frac{57}{79} = \frac{57}{79} \times 100\% = \frac{5700}{79}\%$$

$= \mathbf{72.15\%}$ correct to 2 decimal places.

$$49/67 = \frac{49}{67} = \frac{49}{67} \times 100\% = \frac{4900}{67}\%$$

$= \mathbf{73.13\%}$ correct to 2 decimal places.

Hence, **the second test is marginally better than the first test.**

This question demonstrates how much easier it is to compare two fractions when they are expressed as percentages.

Problem 24. Express 75% as a fraction.

$$75\% = \frac{75}{100} = \frac{3}{4}$$

The fraction $\frac{75}{100}$ is reduced to its simplest form by cancelling, i.e. dividing numerator and denominator by 25.

Problem 25. Express 37.5% as a fraction.

$$37.5\% = \frac{37.5}{100}$$

$$= \frac{375}{1000}$$ by multiplying numerator and denominator by 10

$$= \frac{15}{40}$$ by dividing numerator and denominator by 25

$$= \mathbf{\frac{3}{8}}$$ by dividing numerator and denominator by 5

Problem 26. Find 27% of £65.

$$27\% \text{ of } £65 = \frac{27}{100} \times 65 = \mathbf{£17.55} \text{ by calculator}$$

Problem 27. A 160 GB iPod is advertised as costing £190 excluding VAT. If VAT is added at 20%, what will be the total cost of the iPod?

$$\text{VAT} = 20\% \text{ of } £190 = \frac{20}{100} \times 190 = £38$$

$$\text{Total cost of iPod} = £190 + £38 = \mathbf{£228}$$

A quicker method to determine the total cost is: $1.20 \times £190 = \mathbf{£228}$

Problem 28. Express 23 cm as a percentage of 72 cm, correct to the nearest 1%.

$$23 \text{ cm as a percentage of } 72 \text{ cm} = \frac{23}{72} \times 100\%$$

$$= 31.94444...\%$$

$$= \mathbf{32\%} \text{ correct to the nearest } 1\%$$

Problem 29. A box of screws increases in price from £45 to £52. Calculate the percentage change in cost, correct to 3 significant figures.

$$\% \text{ change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

$$= \frac{52 - 45}{45} \times 100\% = \frac{7}{45} \times 100$$

$$= \mathbf{15.6\%} = \textbf{percentage change in cost}$$

Problem 30. A drilling speed should be set to 400 rev/min. The nearest speed available on the machine is 412 rev/min. Calculate the percentage over-speed.

% over-speed

$$= \frac{\text{available speed} - \text{correct speed}}{\text{correct speed}} \times 100\%$$

$$= \frac{412 - 400}{400} \times 100\%$$

$$= \frac{12}{400} \times 100\% = \mathbf{3\%}$$

Now try the following Practice Exercise

Practice Exercise 7 Percentages

In problems 1 and 2, express the given numbers as percentages.

1. 0.057 [5.7%]
2. 0.374 [37.4%]
3. Express 20% as a decimal number [0.20]
4. Express $\frac{11}{16}$ as a percentage [68.75%]
5. Express $\frac{5}{13}$ as a percentage, correct to 3 decimal places [38.462%]
6. Place the following in order of size, the smallest first, expressing each as percentages, correct to 1 decimal place:
   (a) $\frac{12}{21}$ (b) $\frac{9}{17}$ (c) $\frac{5}{9}$ (d) $\frac{6}{11}$
   [(b) 52.9%, (d) 54.5%, (c) 55.6%, (a) 57.1%]
7. Express 65% as a fraction in its simplest form $\left[\frac{13}{20}\right]$
8. Calculate 43.6% of 50 kg [21.8 kg]
9. Determine 36% of 27 m [9.72 m]
10. Calculate correct to 4 significant figures:
    (a) 18% of 2758 tonnes
    (b) 47% of 18.42 grams
    (c) 147% of 14.1 seconds
    [(a) 496.4 t (b) 8.657 g (c) 20.73 s]

11. Express: (a) 140 kg as a percentage of 1 t (b) 47 s as a percentage of 5 min (c) 13.4 cm as a percentage of 2.5 m
[(a) 14% (b) 15.67% (c) 5.36%]

12. A computer is advertised on the internet at £520, exclusive of VAT. If VAT is payable at 20%, what is the total cost of the computer? [£624]

13. Express 325 mm as a percentage of 867 mm, correct to 2 decimal places. [37.49%]

14. When signing a new contract, a Premiership footballer's pay increases from £15,500 to £21,500 per week. Calculate the percentage pay increase, correct to 3 significant figures. [38.7%]

15. A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length. [2.7%]

1.8 Laws of indices

The manipulation of indices, powers and roots is a crucial underlying skill needed in algebra.

Law 1: When multiplying two or more numbers having the same base, the indices are added.

For example, $2^2 \times 2^3 = 2^{2+3} = 2^5$

and $5^4 \times 5^2 \times 5^3 = 5^{4+2+3} = 5^9$

More generally, $\boldsymbol{a^m \times a^n = a^{m+n}}$

For example, $a^3 \times a^4 = a^{3+4} = a^7$

Law 2: When dividing two numbers having the same base, the index in the denominator is subtracted from the index in the numerator.

For example, $\frac{2^5}{2^3} = 2^{5-3} = 2^2$

and $\frac{7^8}{7^5} = 7^{8-5} = 7^3$

More generally, $\boldsymbol{\frac{a^m}{a^n} = a^{m-n}}$

For example, $\frac{c^5}{c^2} = c^{5-2} = c^3$

Law 3: When a number which is raised to a power is raised to a further power, the indices are multiplied.

For example, $\left(2^2\right)^3 = 2^{2\times 3} = 2^6$

and $\left(3^4\right)^2 = 3^{4\times 2} = 3^8$

More generally, $\boldsymbol{(a^m)^n = a^{mn}}$

For example, $\left(d^2\right)^3 = d^{2\times 3} = d^6$

Law 4: When a number has an index of 0, its value is 1.

For example, $3^0 = 1$

and $17^0 = 1$

More generally, $\boldsymbol{a^0 = 1}$

Law 5: A number raised to a negative power is the reciprocal of that number raised to a positive power.

For example, $3^{-4} = \frac{1}{3^4}$ and $\frac{1}{2^{-3}} = 2^3$

More generally, $\boldsymbol{a^{-n} = \frac{1}{a^n}}$

For example, $a^{-2} = \frac{1}{a^2}$

Law 6: When a number is raised to a fractional power the denominator of the fraction is the root of the number and the numerator is the power.

For example, $8^{\frac{2}{3}} = \sqrt[3]{8^2} = (2)^2 = 4$

and $25^{\frac{1}{2}} = \sqrt[2]{25^1} = \sqrt{25^1}$

$= \pm 5$ (Note that $\sqrt{\ } \equiv \sqrt[2]{\ }$)

More generally, $\boldsymbol{a^{\frac{m}{n}} = \sqrt[n]{a^m}}$

For example, $x^{\frac{4}{3}} = \sqrt[3]{x^4}$

Problem 31. Evaluate in index form $5^3 \times 5 \times 5^2$

$5^3 \times 5 \times 5^2 = 5^3 \times 5^1 \times 5^2$ (Note that 5 means 5^1)

$= 5^{3+1+2} = \boldsymbol{5^6}$ from law 1

Problem 32. Evaluate $\frac{3^5}{3^4}$

From law 2: $\dfrac{3^5}{3^4} = 3^{5-4} = 3^1 = \mathbf{3}$

Problem 33. Evaluate $\dfrac{2^4}{2^4}$

$$\frac{2^4}{2^4} = 2^{4-4} \quad \text{from law 2}$$
$$= 2^0 = \mathbf{1} \quad \text{from law 4}$$

Any number raised to the power of zero equals 1

Problem 34. Evaluate $\dfrac{3 \times 3^2}{3^4}$

$$\frac{3 \times 3^2}{3^4} = \frac{3^1 \times 3^2}{3^4} = \frac{3^{1+2}}{3^4} = \frac{3^3}{3^4}$$
$$= 3^{3-4} = 3^{-1} \quad \text{from laws 1 and 2}$$
$$= \mathbf{\frac{1}{3}} \quad \text{from law 5}$$

Problem 35. Evaluate $\dfrac{10^3 \times 10^2}{10^8}$

$$\frac{10^3 \times 10^2}{10^8} = \frac{10^{3+2}}{10^8} = \frac{10^5}{10^8} \quad \text{from law 1}$$
$$= 10^{5-8} = 10^{-3} \quad \text{from law 2}$$
$$= \frac{1}{10^{+3}} = \frac{1}{1000} \quad \text{from law 5}$$

Hence, $\dfrac{10^3 \times 10^2}{10^8} = \mathbf{10^{-3}} = \mathbf{\dfrac{1}{1000}} = \mathbf{0.001}$

Problem 36. Simplify: (a) $(2^3)^4$ (b) $(3^2)^5$ expressing the answers in index form.

From law 3: (a) $(2^3)^4 = 2^{3\times 4} = \mathbf{2^{12}}$

(b) $(3^2)^5 = 3^{2\times 5} = \mathbf{3^{10}}$

Problem 37. Evaluate $\dfrac{(10^2)^3}{10^4 \times 10^2}$

From laws 1, 2, and 3: $\dfrac{(10^2)^3}{10^4 \times 10^2} = \dfrac{10^{(2\times 3)}}{10^{(4+2)}}$

$$= \frac{10^6}{10^6} = 10^{6-6}$$
$$= 10^0 = \mathbf{1}$$

Problem 38. Evaluate: (a) $4^{1/2}$ (b) $16^{3/4}$ (c) $27^{2/3}$ (d) $9^{-1/2}$

(a) $4^{1/2} = \sqrt{4} = \mathbf{\pm 2}$

(b) $16^{3/4} = \sqrt[4]{16^3} = (2)^3 = \mathbf{8}$

(Note that it does not matter whether the 4th root of 16 is found first or whether 16 cubed is found first; the same answer will result)

(c) $27^{2/3} = \sqrt[3]{27^2} = (3)^2 = \mathbf{9}$

(d) $9^{-1/2} = \dfrac{1}{9^{1/2}} = \dfrac{1}{\sqrt{9}} = \dfrac{1}{\pm 3} = \mathbf{\pm\dfrac{1}{3}}$

Problem 39. Simplify $a^2b^3c \times ab^2c^5$

$$a^2b^3c \times ab^2c^5 = a^2 \times b^3 \times c \times a \times b^2 \times c^5$$
$$= a^2 \times b^3 \times c^1 \times a^1 \times b^2 \times c^5$$

Grouping together like terms gives:

$$a^2 \times a^1 \times b^3 \times b^2 \times c^1 \times c^5$$

Using law 1 of indices gives:

$$a^{2+1} \times b^{3+2} \times c^{1+5} = a^3 \times b^5 \times c^6$$

i.e. $\mathbf{a^2b^3c \times ab^2c^5 = a^3b^5c^6}$

Problem 40. Simplify $\dfrac{x^5y^2z}{x^2yz^3}$

$$\frac{x^5y^2z}{x^2yz^3} = \frac{x^5 \times y^2 \times z}{x^2 \times y \times z^3} = \frac{x^5}{x^2} \times \frac{y^2}{y^1} \times \frac{z}{z^3}$$
$$= x^{5-2} \times y^{2-1} \times z^{1-3} \quad \text{by law 2}$$
$$= x^3 \times y^1 \times z^{-2} = \mathbf{x^3yz^{-2}} \quad \text{or} \quad \mathbf{\frac{x^3y}{z^2}}$$

Now try the following Practice Exercise

Practice Exercise 8 Laws of indices

In questions 1 to 18, evaluate without the aid of a calculator

1. Evaluate $2^2 \times 2 \times 2^4$ [$2^7 = 128$]
2. Evaluate $3^5 \times 3^3 \times 3$ in index form [3^9]
3. Evaluate $\dfrac{2^7}{2^3}$ [$2^4 = 16$]
4. Evaluate $\dfrac{3^3}{3^5}$ $\left[3^{-2} = \dfrac{1}{3^2} = \dfrac{1}{9}\right]$
5. Evaluate 7^0 [1]

6. Evaluate $\dfrac{2^3 \times 2 \times 2^6}{2^7}$ [$2^3 = 8$]

7. Evaluate $\dfrac{10 \times 10^6}{10^5}$ [$10^2 = 100$]

8. Evaluate $10^4 \div 10$ [$10^3 = 1000$]

9. Evaluate $\dfrac{10^3 \times 10^4}{10^9}$ $\left[10^{-2} = \dfrac{1}{10^2} = \dfrac{1}{100} = 0.01\right]$

10. Evaluate $5^6 \times 5^2 \div 5^7$ [5]

11. Evaluate $(7^2)^3$ in index form [7^6]

12. Evaluate $(3^3)^2$ [$3^6 = 729$]

13. Evaluate $\dfrac{3^7 \times 3^4}{3^5}$ in index form [3^6]

14. Evaluate $\dfrac{(9 \times 3^2)^3}{(3 \times 27)^2}$ in index form [3^4]

15. Evaluate $\dfrac{(16 \times 4)^2}{(2 \times 8)^3}$ [1]

16. Evaluate $\dfrac{5^{-2}}{5^{-4}}$ [$5^2 = 25$]

17. Evaluate $\dfrac{3^2 \times 3^{-4}}{3^3}$ $\left[3^{-5} = \dfrac{1}{3^5} = \dfrac{1}{243}\right]$

18. Evaluate $\dfrac{7^2 \times 7^{-3}}{7 \times 7^{-4}}$ [$7^2 = 49$]

In problems 19 to 36, simplify the following, giving each answer as a power

19. $z^2 \times z^6$ [z^8]

20. $a \times a^2 \times a^5$ [a^8]

21. $n^8 \times n^{-5}$ [n^3]

22. $b^4 \times b^7$ [b^{11}]

23. $b^2 \div b^5$ $\left[b^{-3} \text{ or } \dfrac{1}{b^3}\right]$

24. $c^5 \times c^3 \div c^4$ [c^4]

25. $\dfrac{m^5 \times m^6}{m^4 \times m^3}$ [m^4]

26. $\dfrac{(x^2)(x)}{x^6}$ $\left[x^{-3} \text{ or } \dfrac{1}{x^3}\right]$

27. $\left(x^3\right)^4$ [x^{12}]

28. $\left(y^2\right)^{-3}$ $\left[y^{-6} \text{ or } \dfrac{1}{y^6}\right]$

29. $\left(t \times t^3\right)^2$ [t^8]

30. $\left(c^{-7}\right)^{-2}$ [c^{14}]

31. $\left(\dfrac{a^2}{a^5}\right)^3$ $\left[a^{-9} \text{ or } \dfrac{1}{a^9}\right]$

32. $\left(\dfrac{1}{b^3}\right)^4$ $\left[\dfrac{1}{b^{12}} \text{ or } b^{-12}\right]$

33. $\left(\dfrac{b^2}{b^7}\right)^{-2}$ [b^{10}]

34. $\dfrac{1}{\left(s^3\right)^3}$ $\left[\dfrac{1}{s^9} \text{ or } s^{-9}\right]$

35. $p^3qr^2 \times p^2q^5r \times pqr^2$ [$p^6q^7r^5$]

36. $\dfrac{x^3y^2z}{x^5yz^3}$ $\left[x^{-2}yz^{-2} \text{ or } \dfrac{y}{x^2z^2}\right]$

1.9 Simultaneous equations

The solution of simultaneous equations is demonstrated in the following worked problems.

Problem 41. If 6 apples and 2 pears cost £1.80 and 8 apples and 6 pears cost £2.90, calculate how much an apple and a pear each cost.

Let an apple = A and a pear = P, then:

$$6A + 2P = 180 \qquad (1)$$

$$8A + 6P = 290 \qquad (2)$$

From equation (1), $6A = 180 - 2P$

and $$A = \frac{180 - 2P}{6} = 30 - 0.3333P \qquad (3)$$

From equation (2), $8A = 290 - 6P$

and $$A = \frac{290 - 6P}{8} = 36.25 - 0.75P \qquad (4)$$

Equating (3) and (4) gives:

$$30 - 0.3333P = 36.25 - 0.75P$$

i.e. $0.75P - 0.3333P = 36.25 - 30$

and $0.4167P = 6.25$

and $P = \frac{6.25}{0.4167} = 15$

Substituting in (3) gives: $A = 30 - 0.3333(15)$
$= 30 - 5 = 25$

Hence, **an apple costs 25p and a pear costs 15p**
The above method of solving simultaneous equations is called the **substitution method**.

Problem 42. If 6 bananas and 5 peaches cost £3.45 and 4 bananas and 8 peaches cost £4.40, calculate how much a banana and a peach each cost.

Let a banana = B and a peach = P, then:

$$6B + 5P = 345 \quad (1)$$

$$4B + 8P = 440 \quad (2)$$

Multiplying equation (1) by 2 gives:

$$12B + 10P = 690 \quad (3)$$

Multiplying equation (2) by 3 gives:

$$12B + 24P = 1320 \quad (4)$$

Equation (4) – equation (3) gives: $14P = 630$

from which, $P = \frac{630}{14} = 45$

Substituting in (1) gives: $6B + 5(45) = 345$

i.e. $6B = 345 - 5(45)$

i.e. $6B = 120$

and $B = \frac{120}{6} = 20$

Hence, **a banana costs 20p and a peach costs 45p**
The above method of solving simultaneous equations is called the **elimination method**.

Problem 43. If 20 bolts and 2 spanners cost £10, and 6 spanners and 12 bolts cost £18, how much does a spanner and a bolt cost?

Let s = a spanner and b = a bolt.

Therefore, $2s + 20b = 10 \quad (1)$

and $6s + 12b = 18 \quad (2)$

Multiplying equation (1) by 3 gives:

$$6s + 60b = 30 \quad (3)$$

Equation (3) – equation (2) gives: $48b = 12$

from which, $b = \frac{12}{48} = 0.25$

Substituting in (1) gives: $2s + 20(0.25) = 10$

i.e. $2s = 10 - 20(0.25)$

i.e. $2s = 5$

and $s = \frac{5}{2} = 2.5$

Therefore, **a spanner costs £2.50 and a bolt costs £0.25 or 25p**

Now try the following Practice Exercises

Practice Exercise 9 Simultaneous equations

1. If 5 apples and 3 bananas cost £1.45 and 4 apples and 6 bananas cost £2.42, determine how much an apple and a banana each cost. [apple = 8p, banana = 35p]

2. If 7 apples and 4 oranges cost £2.64 and 3 apples and 3 oranges cost £1.35, determine how much an apple and an orange each cost. [apple = 28p, orange = 17p]

3. Three new cars and four new vans supplied to a dealer together cost £93000, and five new cars and two new vans of the same models cost £99000. Find the respective costs of a car and a van. [car = £15000, van = £12000]

4. In a system of forces, the relationship between two forces F_1 and F_2 is given by:
$$5F_1 + 3F_2 = -6$$
$$3F_1 + 5F_2 = -18$$
Solve for F_1 and F_2
[$F_1 = 1.5, F_2 = -4.5$]

5. Solve the simultaneous equations:
$a + b = 7$
$a - b = 3$ [$a = 5, b = 2$]

6. Solve the simultaneous equations:
$8a - 3b = 51$
$3a + 4b = 14$ [$a = 6$, $b = -1$]

Practice Exercise 10 Multiple-choice questions on revisionary mathematics

(Answers on page 335)

1. 73° is equivalent to:
(a) 23.24 rad (b) 1.274 rad
(c) 0.406 rad (d) 4183 rad

2. 0.52 radians is equivalent to:
(a) 93.6° (b) 0.0091°
(c) 1.63° (d) 29.79°

3. $3\pi/4$ radians is equivalent to:
(a) 135° (b) 270°
(c) 45° (d) 67.5°

4. In the right-angled triangle *ABC* shown in Figure 1.8, sine *A* is given by:
(a) b/a (b) c/b
(c) b/c (d) a/b

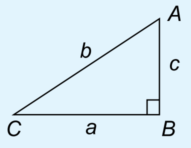


Figure 1.8

5. In the right-angled triangle *ABC* shown in Figure 1.8, cosine *C* is given by:
(a) a/b (b) c/b
(c) a/c (d) b/a

6. In the right-angled triangle *ABC* shown in Figure 1.8, tangent *A* is given by:
(a) b/c (b) a/c
(c) a/b (d) c/a

7. In the right-angled triangle *PQR* shown in Figure 1.9, angle *R* is equal to:
(a) 41.41° (b) 48.59°
(c) 36.87° (d) 53.13°

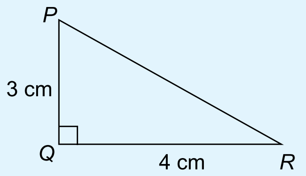


Figure 1.9

8. In the triangle *ABC* shown in Figure 1.10, side '*a*' is equal to:
(a) 61.27 mm
(b) 86.58 mm
(c) 96.41 mm
(d) 54.58 mm

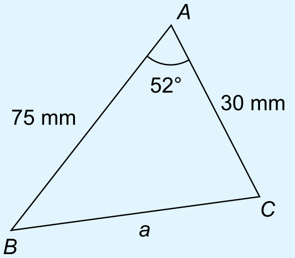


Figure 1.10

9. In the triangle *ABC* shown in Figure 1.10, angle *B* is equal to:
(a) 0.386° (b) 22.69°
(c) 74.71° (d) 23.58°

10. Removing the brackets from the expression: $a[b + 2c - d\{(e - f) - g(m - n)\}]$ gives:
(a) $ab + 2ac - ade - adf + adgm - adgn$
(b) $ab + 2ac - ade - adf - adgm - adgn$
(c) $ab + 2ac - ade + adf + adgm - adgn$
(d) $ab + 2ac - ade - adf + adgm + adgn$

11. $\frac{5}{6} + \frac{1}{5} - \frac{2}{3}$ is equal to:
(a) $\frac{1}{2}$ (b) $\frac{11}{30}$
(c) $-\frac{1}{2}$ (d) $1\frac{7}{10}$

12. $1\frac{1}{3}+1\frac{2}{3}\div 2\frac{2}{3}-\frac{1}{3}$ is equal to:

(a) $1\frac{2}{7}$ (b) $\frac{19}{24}$

(c) $2\frac{1}{21}$ (d) $1\frac{5}{8}$

13. $\frac{3}{4}\div 1\frac{3}{4}$ is equal to:

(a) $\frac{3}{7}$ (b) $1\frac{9}{16}$

(c) $1\frac{5}{16}$ (d) $2\frac{1}{2}$

14. 11 mm expressed as a percentage of 41 mm is:

(a) 2.68, correct to 3 significant figures
(b) 2.6, correct to 2 significant figures
(c) 26.83, correct to 2 decimal places
(d) 0.2682, correct to 4 decimal places

15. The value of $\frac{2^{-3}}{2^{-4}}-1$ is equal to:

(a) 1 (b) 2

(c) $-\frac{1}{2}$ (d) $\frac{1}{2}$

16. In an engineering equation $\frac{3^4}{3^r}=\frac{1}{9}$. The value of r is:

(a) – 6 (b) 2
(c) 6 (d) – 2

17. $16^{-\frac{3}{4}}$ is equal to:

(a) 8 (b) $-\frac{1}{2^3}$

(c) 4 (d) $\frac{1}{8}$

18. The engineering expression $\frac{(16\times 4)^2}{(8\times 2)^4}$ is equal to:

(a) 4 (b) 2^{-4}

(c) $\frac{1}{2^2}$ (d) 1

19. $(16^{-\frac{1}{4}}-27^{-\frac{2}{3}})$ is equal to:

(a) $\frac{7}{18}$ (b) – 7

(c) $1\frac{8}{9}$ (d) $-8\frac{1}{2}$

20. The solution of the simultaneous equations: $3a-2b=13$ and $2a+5b=-4$ is:

(a) $a=-2, b=3$
(b) $a=1, b=-5$
(c) $a=3, b=-2$
(d) $a=-7, b=2$

References

There are many aspects of mathematics needed in engineering studies; a few have been covered in this chapter. For further engineering mathematics, see the following references:

[1] BIRD J. O. *Basic Engineering Mathematics 6th Edition*, Taylor & Francis, 2014
[2] BIRD J. O. *Engineering Mathematics 7th Edition*, Taylor & Francis, 2014

For fully worked solutions to each of the problems in Practice Exercises 1 to 10 in this chapter, go to the website:
www.routledge.com/cw/bird

Revision Test 1 Revisionary mathematics

This Revision Test covers the material contained in Chapter 1. *The marks for each question are shown in brackets at the end of each question.*

1. Convert, correct to 2 decimal places:
   (a) 76.8° to radians
   (b) 1.724 radians to degrees (4)

2. In triangle *JKL* in Figure RT1.1, find:
   (a) length *KJ* correct to 3 significant figures
   (b) sin *L* and tan *K*, each correct to 3 decimal places

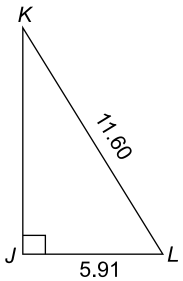


Figure RT1.1

(4)

3. In triangle *PQR* in Figure RT1.2, find angle *P* in decimal form, correct to 2 decimal places

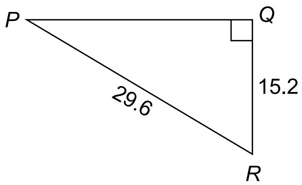


Figure RT1.2

(2)

4. In triangle *ABC* in Figure RT1.3, find lengths *AB* and *AC*, correct to 2 decimal places

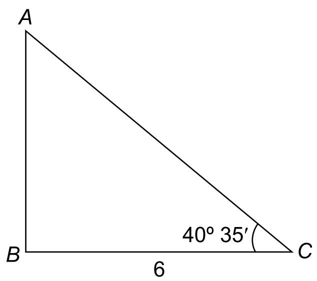


Figure RT1.3

(4)

5. A triangular plot of land *ABC* is shown in Figure RT1.4. Solve the triangle and determine its area

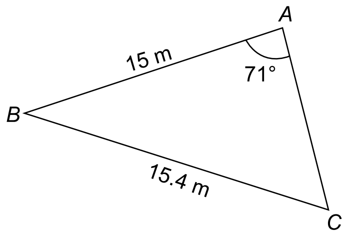


Figure RT1.4

(9)

6. Figure RT1.5 shows a roof truss *PQR* with rafter *PQ* = 3 m. Calculate the length of (a) the roof rise *PP′* (b) rafter PR, and (c) the roof span *QR*. Find also (d) the cross-sectional area of the roof truss

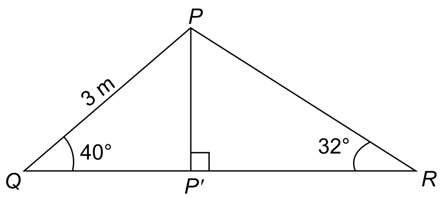


Figure RT1.5

(10)

7. Solve triangle *ABC* given $b = 10$ cm, $c = 15$ cm and $\angle A = 60°$. (7)

8. Remove the brackets and simplify
   $2(3x - 2y) - (4y - 3x)$ (3)

9. Remove the brackets and simplify
   $10a - [3(2a - b) - 4(b - a) + 5b]$ (4)

10. Determine, correct to 2 decimal places, 57% of 17.64 *g* (2)

11. Express 54.7 mm as a percentage of 1.15 m, correct to 3 significant figures. (3)

12. Simplify:
    (a) $\frac{3}{4} - \frac{7}{15}$ (b) $1\frac{5}{8} - 2\frac{1}{3} + 3\frac{5}{6}$ (8)

13. Use a calculator to evaluate:

(a) $1\frac{7}{9} \times \frac{3}{8} \times 3\frac{3}{5}$

(b) $6\frac{2}{3} \div 1\frac{1}{3}$

(c) $1\frac{1}{3} \times 2\frac{1}{5} \div \frac{2}{5}$ (10)

14. Evaluate:

(a) $3 \times 2^3 \times 2^2$

(b) $49^{\frac{1}{2}}$ (4)

15. Evaluate:

(a) $\frac{2^7}{2^2}$ (b) $\frac{10^4 \times 10 \times 10^5}{10^6 \times 10^2}$ (4)

16. Evaluate:

(a) $\frac{2^3 \times 2 \times 2^2}{2^4}$

(b) $\frac{\left(2^3 \times 16\right)^2}{\left(8 \times 2\right)^3}$

(c) $\left(\frac{1}{4^2}\right)^{-1}$ (7)

17. Evaluate:

(a) $(27)^{-\frac{1}{3}}$ (b) $\frac{\left(\frac{2}{3}\right)^{-2} - \frac{2}{9}}{\left(\frac{2}{3}\right)^2}$ (5)

18. Solve the simultaneous equations:

(a) $2x + y = 6$
$5x - y = 22$

(b) $4x - 3y = 11$
$3x + 5y = 30$ (10)

For lecturers/instructors/teachers, fully worked solutions to each of the problems in Revision Test 1, together with a full marking scheme, are available at the website:
www.routledge.com/cw/bird

Chapter 2

Further revisionary mathematics

***Why it is important to understand:* Further revisionary mathematics**

In engineering there are many different quantities to get used to, and hence many units to become familiar with. For example, force is measured in newtons, electric current is measured in amperes and pressure is measured in pascals. Sometimes the units of these quantities are either very large or very small and hence prefixes are used. For example, 1000 pascals may be written as 10^3 Pa which is written as 1 kPa in prefix form, the k being accepted as a symbol to represent 1000 or 10^3. Studying, or working, in an engineering discipline, you very quickly become familiar with the standard units of measurement, the prefixes used and engineering notation. An electronic calculator is extremely helpful with engineering notation.

Most countries have used the metric system of units for many years; however, there are other countries, such as the USA, who still use the imperial system. Hence, metric to imperial unit conversions, and vice-versa, are internationally important and are contained in this chapter.

Graphs have a wide range of applications in engineering and in physical sciences because of their inherent simplicity. A graph can be used to represent almost any physical situation involving discrete objects and the relationship among them. If two quantities are directly proportional and one is plotted against the other, a straight line is produced. Examples of this include an applied force on the end of a spring plotted against spring extension, the speed of a flywheel plotted against time, and strain in a wire plotted against stress (Hooke's law). In engineering, the straight line graph is the most basic graph to draw and evaluate.

There are many practical situations engineers have to analyse which involve quantities that are varying. Typical examples include the stress in a loaded beam, the temperature of an industrial chemical, the rate at which the speed of a vehicle is increasing or decreasing, the current in an electrical circuit or the torque on a turbine blade. Differential calculus, or differentiation, is a mathematical technique for analysing the way in which functions change. This chapter explains how to differentiate the five most common functions. Engineering is all about problem solving and many problems in engineering can be solved using calculus. Physicists, chemists, engineers, and many other scientific and technical specialists use calculus in their everyday work; it is a technique of fundamental importance. Integration has numerous applications in engineering and science and some typical examples include determining areas, mean and r.m.s. values, volumes of solids of revolution, centroids, second moments of area and differential equations. Standard integrals are covered in this chapter, and for any further studies in engineering, differential and integral calculus are unavoidable.

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